

12.6

Add and Subtract Rational Expressions

Goal • Add and subtract rational expressions.

Your Notes

REMEMBER
 You NEED AN LCD TO ADD/SUBTRACT FRACTIONS!

VOCABULARY

Least common denominator of rational expressions (LCD) ① FACTOR EACH DENOMINATOR.
 ② THE LCD IS THE PRODUCT OF ALL THE FACTORS WITH EACH COMMON FACTOR USED ONLY ONCE.
 * SEE EXAMPLE 2

ADDING AND SUBTRACTING RATIONAL EXPRESSIONS WITH THE SAME DENOMINATOR

Let a , b , and c be polynomials where $c \neq 0$.

Algebra

$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$$

$$\frac{a}{c} - \frac{b}{c} = \frac{a-b}{c}$$

TRY THESE

① $\frac{3}{4} + \frac{2}{4} = \frac{5}{4}$

LEAVE ANSWERS AS IMPROPER FRACTIONS

② $\frac{3}{50} - \frac{1}{50} = \frac{2}{50}$
 $\frac{1}{25}$

Reduce FRACTIONS

③ $\frac{5}{6} - \frac{1}{2}(\frac{3}{3}) =$
 $\frac{5}{6} - \frac{3}{6} = \frac{2}{6}$
 $\frac{1}{3}$

LCD = 6

Example 1 Add and subtract with the same denominator

a. $\frac{3}{8x} + \frac{4}{8x} = \frac{3+4}{8x}$
 Common Denom. = $\frac{7}{8x}$

Add numerators.

Simplify.

b. $\frac{2x+9}{x+1} - \frac{7}{x+1} = \frac{2x+9-7}{x+1}$
 Common Denom. = $\frac{2x+2}{x+1}$

Subtract numerators.

Simplify. Reduce

$$= \frac{2(x+1)}{(x+1)}$$

$$= \frac{2}{1}$$

Factor and divide out common factor.

Simplify.

Checkpoint Find the sum or difference.

<p>1. $\frac{x+8}{4x} + \frac{3}{4x}$</p> <div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 10px auto;"> $\frac{x+11}{4x}$ </div>	<p>2. $\frac{6x-5}{x} - \frac{(2x-5)}{x}$ subtract</p> $\frac{6x-5-2x+5}{x} = \frac{4x}{x} = 4$
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Example 3 Add expressions with different denominators

Find the sum $\frac{1}{3x^3} + \frac{5}{4x^4}$

$LCD = 12x^4$

↑ For variables take the highest exponent

Solution

$$\frac{1}{3x^3} + \frac{5}{4x^4}$$

$$= \frac{1 \cdot \boxed{4x}}{3x^3 \cdot \boxed{4x}} + \frac{5 \cdot \boxed{3}}{4x^4 \cdot \boxed{3}}$$

Rewrite fractions using LCD,

$LCD = 12x^4$

$$= \frac{\boxed{4x}}{\boxed{12x^4}} + \frac{\boxed{15}}{\boxed{12x^4}}$$

Simplify numerators and denominators.

← Add fractions.

$$\frac{4x+15}{12x^4}$$

Cannot simplify terms

There are NO common factors
We can not factor anything.

Remember only cancel "FACTORS"!
NOT "TERMS" WITH
ADDITION OR SUBTRACTION!

Your Notes

$$\frac{1+2}{1+3} = \frac{3}{4}$$

$$\frac{1+2}{1+3} \overset{NO}{=} \overset{NO}{=} \text{?}$$

$$\frac{2 \cdot 3}{2 \cdot 5} = \frac{3}{5}$$

Checkpoint Find the sum or difference.

LCD = (x-3)

(A) $\frac{9}{x-3} - \frac{15}{x-3} = \frac{-6}{x-3}$

LCD = (x+4)

(B) $\frac{12}{x+4} + \frac{3x}{x+4} = \frac{3x+12}{x+4}$ ← FACTOR

$\frac{3(x+4)}{(x+4)} = 3$

LCD = 3x

(C) $\frac{10}{3x} - \frac{5}{x} \left(\frac{3}{3}\right) =$

$\frac{10}{3x} - \frac{15}{3x} =$

$\frac{-5}{3x}$

LCD = 2x²

(D) $\left(\frac{x}{x}\right) \frac{25}{2x} + \frac{10}{x^2} \left(\frac{2}{2}\right)$

$\frac{25x}{2x^2} + \frac{20}{2x^2} =$

Simplified form →

$\frac{25x+20}{2x^2}$

Factored Form →

$\frac{5(5x+4)}{2x^2}$

FACTOR TO SEE IF WE CAN REDUCE/SIMPLIFY FRACTIONS.

BOTH ANSWERS ARE CORRECT

ADVANCED TOPICS

Notes

(time permitting)

Example 2 Find the LCD of rational expressions

Find the LCD of the rational expressions.

a. $\frac{1}{3x^3}, \frac{5}{4x^4}$

b. $\frac{7}{x^2 - 4}, \frac{x + 3}{x^2 + x - 2}$

Solution

a. Find the least common multiple of $3x^3$ and $4x^4$.

$3x^3 = 3 \cdot x \cdot x \cdot x$

$4x^4 = 2 \cdot 2 \cdot x \cdot x \cdot x \cdot x$

LCM = $x \cdot x \cdot x \cdot 3 \cdot 2 \cdot 2 \cdot x = 12x^4$

Completely factor

← EVERY FACTOR MUST BE REPRESENTED

The LCD of $\frac{1}{3x^3}$ and $\frac{5}{4x^4}$ is $12x^4$.

b. Find the LCM = LCD of $x^2 - 4$ and $x^2 + x - 2$.

$x^2 - 4 = (x-2)(x+2)$

$x^2 + x - 2 = (x-1)(x+2)$

LCM = $(x+2)(x-2)(x-1)$

← STEP 2: IDENTIFY FACTORS
EVERY FACTOR MUST BE REPRESENTED ONCE

The LCD of $\frac{7}{x^2 - 4}$ and $\frac{x + 3}{x^2 + x - 2}$ is _____.

LCD →

Check

$3x^3 \cdot 4x = 12x^4$

$4x^4 \cdot 3 = 12x^4$

STEP 1: FACTOR

Checkpoint Find the LCD of the rational expressions.

<p>3. $\frac{5}{36x}, \frac{x+2}{4x^3}$ $\frac{1}{36} + \frac{1}{4}x^4$</p> <div style="border: 1px solid purple; padding: 5px; display: inline-block; margin-top: 10px;"> <p style="color: purple; font-size: 1.2em;">LCD = $36x^3$</p> </div>	<p>4. $\frac{7x}{x-8}, \frac{x-1}{x+3}$</p> <div style="border: 1px solid green; padding: 5px; display: inline-block; margin-top: 10px;"> <p style="color: green; font-size: 1.2em;">LCD = $(x-8)(x+3)$</p> </div>
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LCD
① For variable take the highest exp

② you know what to do with numbers

ADD

$\frac{1}{4} + \frac{1}{5} =$

LCD = 20

Your Notes

Example 3

Add expressions with different denominators

Find the sum $\frac{1}{3x^3} + \frac{5}{4x^4}$.

Solution

$$\begin{aligned} & \frac{1}{3x^3} + \frac{5}{4x^4} \\ &= \frac{1 \cdot \boxed{4x}}{3x^3 \cdot \boxed{4x}} + \frac{5 \cdot \boxed{3}}{4x^4 \cdot \boxed{3}} \\ &= \frac{\boxed{4x}}{\boxed{12x^4}} + \frac{\boxed{15}}{\boxed{12x^4}} \\ &= \frac{\boxed{4x + 15}}{\boxed{12x^4}} \end{aligned}$$

$LCD = 12x^4$

Rewrite fractions using LCD,

Simplify numerators and denominators.

Add fractions.

You can add b/c you have can LCD

Example 4

Subtract expressions with different denominators

Find the difference $\frac{x + 1}{x^2 + 5x + 6} - \frac{x - 4}{x^2 - 9}$.

Solution

$$\begin{aligned} & \frac{x + 1}{x^2 + 5x + 6} - \frac{x - 4}{x^2 - 9} \\ &= \frac{x + 1}{(x + 2)(x + 3)} - \frac{x - 4}{(x + 3)(x - 3)} \\ &= \frac{(x + 1)(x - 3)}{(x + 2)(x + 3)(x - 3)} - \frac{(x - 4)(x + 2)}{(x + 3)(x - 3)(x + 2)} \\ &= \frac{(x^2 - 2x - 3) - (x^2 - 2x - 8)}{(x + 2)(x + 3)(x - 3)} \\ &= \frac{\cancel{x^2} - 2\cancel{x} - 3 + \cancel{x^2} + 2\cancel{x} + 8}{(x + 2)(x + 3)(x - 3)} \\ &= \frac{5}{(x + 2)(x + 3)(x - 3)} \end{aligned}$$

STEP I
FACTOR Denominator

STEP II
FIND LCD = (x+2)(x+3)(x-3)

STEP III
Add or Subtract the fractions

STEP IV
Simplify

Change subtract → add then take opposite of all terms behind the - sign

← LEAVE IN FACTORED FORM

Your Notes

✓ **Checkpoint** Find the sum or difference.

(3x+1)

$$5. \frac{9}{x-1} - \frac{15 \cdot (x-1)}{3x+1}$$

$$\frac{27x+9 - (15x-15)}{(x-1)(3x+1)} = \frac{12x+24}{(x-1)(3x+1)}$$

12(x+2)

(x-4)

$$6. \frac{12}{5x} + \frac{3x}{x-4} \quad (5x)$$

$$\frac{12x-48 + 15x^2}{(x-4)(5x)} = \frac{15x^2 + 12x - 48}{5x(x-4)}$$

x-4 IS NOT A FACTOR

$$\begin{array}{r} 15x + 72 \\ x-4 \overline{) 15x^2 + 12x - 48} \\ \underline{-(15x^2 - 60x)} \\ 72x - 48 \\ \underline{-(72x - 288)} \\ 240 \end{array}$$

$$7. \frac{(x-1)(x+1)}{x^2-2x-24} + \frac{4}{x^2-5x-6}$$

$$(x-6)(x+4) \quad (x-6)(x+1)$$

LCD

$$\frac{(x-1)(x+1) + 4(x+4)}{(x-6)(x+4)(x+1)} = \frac{x^2-1 + 4x+16}{(x-6)(x+4)(x+1)} = \frac{x^2+4x+15}{(x-6)(x+4)(x+1)}$$

$$8. \frac{(x+1)(x+2)}{x^2+2x-15} - \frac{(x-6)(x+5)}{x^2+4x-21}$$

$$(x+5)(x-3) \quad (x+7)(x-3)$$

$$\frac{x^2+9x+14 - (x^2-x-30)}{(x+5)(x-3)(x+7)} = \frac{10x+44}{(x+5)(x-3)(x+7)}$$