12.5 Multiply and Divide Rational Expressions

Goal · Multiply and divide rational expressions.

Your Notes

MULTIPLYING AND DIVIDING RATIONAL EXPRESSIONS

Let a, b, c, and d be polynomials.

Algebra

MULTIPLY $\quad \dfrac{a}{b} \cdot \dfrac{c}{d} = \boxed{\dfrac{ac}{bd}}$ where $b \neq 0$ and $d \neq 0$

DIVIDE $\quad \dfrac{a}{b} \div \dfrac{c}{d} = \dfrac{a}{b} \cdot \boxed{\dfrac{d}{c}} = \boxed{\dfrac{ad}{bc}}$ where $b \neq 0$, $c \neq 0$, and $d \neq 0$

Examples

← Factored form

$\dfrac{2x}{x+1} \cdot \dfrac{x}{5} = \boxed{\dfrac{2x \cdot x}{5(x+1)}}$

$\dfrac{3}{x^2} \div \dfrac{x}{5} = \dfrac{3}{x^2} \cdot \boxed{\dfrac{5}{x}} = \boxed{\dfrac{15}{x^3}}$

reciprocal

Simplified form → $\dfrac{2x^2}{5x+5}$

Example 1 · Multiply rational expressions involving monomials

Find the product $\dfrac{3x^4}{4x^3} \cdot \dfrac{2x^2}{5x^3}$.

Solution

$\dfrac{3x^4}{4x^3} \cdot \dfrac{2x^2}{5x^3} = \boxed{\dfrac{6x^6}{20x^6}}$ ① ② Multiply numerators and denominators.

$= \boxed{\dfrac{6}{20}}$ CANCEL

reduce & keep as an improper fraction

$= \boxed{\dfrac{3}{10}}$ Simplify.

OR FOLLOW RULES OF EXPONENTS

$x^4 \cdot x^2 = x^{4+2} = x^6$

$\left(x^4\right)^2 = x^{4 \cdot 2} = x^8$

$\dfrac{x^4}{x^2} = x^{4-2} = x^2$

KEEP ANSWER AS AN IMPROPER FRACTION

EX/ $\dfrac{5}{2} \longrightarrow 2\dfrac{1}{2}$

STEPS

① Completely factor

② Cancel common factors

③ Simplify

Example 2 *Multiply rational expressions involving polynomials*

Find the product $\dfrac{x}{5x^2 - 6x - 8} \cdot \dfrac{2x^2 - 4x}{7x^2}$.

Solution

$$\dfrac{x}{5x^2 - 6x - 8} \cdot \dfrac{2x^2 - 4x}{7x^2} = \dfrac{x \cdot 2x(x-2)}{(5x+4)(x-2) \cdot 7x^2}$$

$$= \dfrac{2}{7(5x+4)}$$ FACTORED FORM

$$\boxed{\dfrac{2}{35x + 28}}$$

Example 3 *Multiply a rational expression by a polynomial*

Find the product $\dfrac{4x}{x^2 - x - 12} \cdot (x - 4)$.

Solution

$$\dfrac{4x}{x^2 - x - 12} \cdot (x - 4)$$

$$= \dfrac{4x}{x^2 - x - 12} \cdot \dfrac{(x-4)}{1}$$ Rewrite polynomial as a fraction.

$$= \dfrac{4 \cdot x \cdot (x-4)}{(x+3)(x-4)}$$ ← Completely factor

Factor and divide out common factor.

$$= \dfrac{4 \cdot x}{(x+3)}$$

$$= \boxed{\dfrac{4x}{x+3}}$$ Simplify. means no ( )'s

✓ **Checkpoint** Find the product.

1. $\dfrac{2x^4}{5x^2} \cdot \dfrac{6x}{3x^3}$ ⟶ $\dfrac{12 x^{\cancel{5}}}{15 \cancel{x} x^5} = \dfrac{12}{15} = \boxed{4/5}$

2. $\dfrac{x^2 - 5x + 4}{3x^2 - 12x} \cdot \dfrac{2x^2 + 2}{x^2 + 6x - 7}$

$\dfrac{(x-4)(x-1) \cdot 2\cancel{\blacksquare}(x^2+1)}{3x(x-4) \cdot (x+7)(x-1)} = \dfrac{2(x^2+1)}{3x(x+7)} = \boxed{\dfrac{2x^2+2}{3x^2+21x}}$

↑ IN FACTORED FORM

↑ COMPLETELY SIMPLIFIED   NO ( )'s

3. $\dfrac{2x}{x^2 + 5x - 24} \cdot \dfrac{(x+8)}{1} = \dfrac{2x\cancel{(x+8)}}{\cancel{(x+8)}(x-3)} = \boxed{\dfrac{2x}{x-3}}$

1	24
2	12
3	8

Example 4 **Divide rational expressions involving polynomials**

Find the quotient $\dfrac{x^2 + 5x - 24}{x^2 + 9x + 8} \div \dfrac{x^2 - 9}{6x - 18}$.

Solution

$\dfrac{x^2 + 5x - 24}{x^2 + 9x + 8} \overset{①}{\div} \dfrac{x^2 - 9}{6x - 18}^{②}$

$= \dfrac{x^2 + 5x - 24}{x^2 + 9x + 8} \cdot \boxed{\dfrac{6x-18}{x^2-9}}$ Multiply by multiplicative inverse.
↳ reciprocal

$= \dfrac{(x+8)(x-3) \cdot 6(x-3)}{(x+8)(x+1) \cdot (x-3)(x+3)}$ ③ Factor and divide out common factors.

$= \boxed{\dfrac{6(x-3)}{(x+1)(x+3)}}$

$= \boxed{\dfrac{6x-18}{x^2+4x+3}}$ ④ Simplify.

FACTORED FORM →

Remember
- FACTOR
- CANCEL
- SIMPLIFY

Review ÷ :

① Change ÷ to ×
② TAKE THE RECIPROCAL OF THE FRACTION AFTER THE ÷ SIGN

Example 5 — *Divide a rational expression by a polynomial*

Find the quotient $\dfrac{x^2 - 25}{x - 3} \div (x - 5)$.

Solution

$\dfrac{x^2 - 25}{x - 3} \div (x - 5)$

$= \dfrac{x^2 - 25}{x - 3} \div \dfrac{\boxed{X-5}}{\boxed{}}$ ① Rewrite polynomial as fraction. **AND PUT POLYNOMIAL OVER 1.**

$= \dfrac{x^2 - 25}{x - 3} \cdot \dfrac{\boxed{1}}{\boxed{X-5}}$ ② Multiply by multiplicative inverse. **(reciprocal)**

$= \dfrac{(X-5)(X+5) \cdot 1}{(X-3)\,(X-5)}$ ~~Multiply numerators and denominators.~~

$= \dfrac{\boxed{}}{\boxed{}}$ ③ Factor and divide out common factors.

$= \boxed{\dfrac{X+5}{X-3}}$ ④ Simplify.

HINT: ALWAYS SHOW THE FACTORED ANSWER

✓ **Checkpoint** Find the quotient.

4. $\dfrac{x^2 + 2x - 15}{x^2 + 4x - 5} \div (7x - 14)$

$\dfrac{(X+5)(X-3)}{(X+5)(X-1)} \cdot \dfrac{7(X-2)}{(X-2)(X+2)} = \dfrac{7(X-3)}{(X-1)(X+2)} = \boxed{\dfrac{7X-21}{X^2+X-2}}$

↑ Completely Simplified

5. $\dfrac{x^2 + 8x + 7}{x^2 - 1} \div (x + 7)$

$\dfrac{(X+7)(X+1)}{(X-1)(X+1)} \cdot \dfrac{1}{X+7} = \boxed{\dfrac{1}{X-1}}$