12.4 Simplify Rational Expressions

(FRACTIONS WITH VARIABLES)

Goal • Simplify rational expressions.

Your Notes

VOCABULARY

Rational expression Can be written as a ratio of 2 POLYNOMIALS. EX] $\dfrac{x^2}{(x+2)(x-1)}$

Excluded value is a number that makes the expression UNDEFINED "ZERO CAN NOT BE IN THE DENOMINATOR"

Simplest form of a rational expression HAS NO FACTORS IN COMMON IN THE NUMERATOR AND DENOMINATOR.

EXAMPLE — Simplify

$$\frac{\cancel{(x+1)}\cdot(x+2)}{(x+4)\cdot\cancel{(x+1)}} = \boxed{\frac{x+2}{x+4}}$$

The Factors are $(x+1)$, $(x+2)$ and $(x+4)$. FACTORS ARE SEPERATED BY MULTIPLICATION.

ARITHMETIC EXAMPLE Simplify

Ⓐ $\dfrac{\cancel{2}\cdot 5}{\cancel{2}\cdot 6} = \boxed{\dfrac{5}{6}}$

Cancel the Common factor 2.

Ⓑ $\dfrac{1+2}{1+3} = \boxed{\dfrac{3}{4}}$

$\dfrac{\cancel{1}+2}{\cancel{1}+3} = \dfrac{2}{3}$ ✗

You can not cancel TERMS 🙂

EXAMPLE

$$\frac{x^2}{(x+2)(x-1)}$$

EVALUATE

IF X = 0

$\dfrac{0^2}{(0+2)(0-1)} = \dfrac{0}{-2} = \boxed{0}$

IF X = 1

$\dfrac{1^2}{(1+2)(1-1)} = \dfrac{1}{3\cdot 0} = \dfrac{1}{0}$

$\boxed{\text{UNDEFINED}}$

IF X = 2

$\dfrac{2^2}{(2+2)(2-1)} = \dfrac{4}{4(1)} = \dfrac{4}{4} = 1$

IF X = -2

$\dfrac{(-2)^2}{(-2+2)(-2-1)} = \dfrac{4}{0(-3)} = \dfrac{4}{0}$

$\boxed{\text{UNDEFINED}}$

∴ 1 and -2 are excluded values

$X \neq 1, -2$

What number(s) make the den. Zero?

Example 1 Find **excluded values**

Find the excluded values, if any, of the expression.

a. $\dfrac{x}{4x - 8}$ $x \neq 2$ $= \dfrac{2}{4(2)-8} = \dfrac{2}{0}$ UNDEFINED

b. $\dfrac{3x}{x^2 - 16}$ $x \neq 4$ $\dfrac{3(4)}{4^2-16} = \dfrac{12}{0}$ AND $x \neq -4$ $\dfrac{3(-4)}{(-4)^2-16} = \dfrac{-12}{0}$

Solution

A SET DEN. TO 0 AND SOLVE FOR X: $4x-8=0$
$\quad +8 \quad +8$
$\dfrac{4x}{4} = \dfrac{8}{4}$
$\boxed{x=2}$

a. The expression $\dfrac{x}{4x - 8} \neq 0$ is undefined when
$4x-8 = 0$, or $x = 2$. The excluded value is __2__. → $\boxed{x \neq 2}$

B FACTOR. Then set each factor to 0 and solve for X.

b. The expression $\dfrac{3x}{x^2 - 16}$ is undefined when
$x^2-16 = 0$, or $(x-4)(x+4) = 0$.
The solutions of the equation are __4__ and __-4__
The excluded values are __-4__ and __4__.
$\boxed{x \neq -4, 4}$

✓ **Checkpoint** Find the **excluded values**, if any, of the expression.

1. $\dfrac{x + 6}{14x}$	2. $\dfrac{9x + 1}{x^2 - x - 20}$

Look AT THE DEN ONLY!

LINEAR so Set =0

$14x = 0$

$\boxed{x \neq 0}$

QE so FACTOR!

$x^2 - x - 20 = 0$

$(x - 5)(x + 4) = 0$

$x - 5 = 0 \qquad x + 4 = 0$

$\boxed{x = 5} \qquad \boxed{x = -4}$

These are the excluded values

$\boxed{x \neq -4, 5}$

Handwritten notes (left margin)

SIMPLIFY FRACTIONS

① Completely factor both the num. and den.

② CANCEL ALL COMMON FACTORS

③ Simplify *

* IF THERE ARE MORE THAN 2 (3+) BINOMIAL THEN LEAVE IN FACTURED FORM

SIMPLIFYING RATIONAL EXPRESSIONS

Let a, b, and c be polynomials where $b \neq 0$ and $c \neq 0$.

Algebra

$$\frac{ac}{bc} = \frac{a \cdot c}{b \cdot c} = \frac{a}{b}$$

Example

$$\frac{3x - 9}{4x - 12} = \frac{3(x - 3)}{4(x - 3)} = \frac{3}{4}$$

Example 2 Simplify expressions by d____

Simplify the rational expression, if possible. State the excluded values.

a. $\dfrac{18x}{6x^2} = \dfrac{6 \cdot 3 \cdot x}{6 \cdot x \cdot x}$ ← Divide out common factors. OR USE Rules of Exponents *

$\ast \quad \Longrightarrow = \dfrac{3}{x}$ Simplify.

The excluded value is $\boxed{x \neq 0}$

b. $\dfrac{12x^2 - 6x}{24x} = \dfrac{6x(2x - 1)}{6x(4)}$ ① Factor numerator and denominator.

$= $ ② Divide out common factors. $6x$

$= \boxed{\dfrac{2x - 1}{4}}$ ③ Simplify. (keep as improper fraction)

The excluded value is $\underline{0}$ because you look at the denominator of the factored form. $(x \neq 0)$

✔ **Checkpoint** Simplify the rational expression, if possible. State the excluded values. Tip: only look at DEN

3. $\dfrac{7}{5x + 3}$ simplified

$5x + 3 = 0$
$\quad -3 \quad -3$
$\dfrac{5x}{5} = \dfrac{-3}{5}$
$\boxed{x \neq -3/5}$

4. $\dfrac{5x}{5x^2 - 125}$

$\dfrac{5x}{5(x^2 - 25)} =$

$\dfrac{5x}{5(x+5)(x-5)}$

$\dfrac{x}{x^2 - 25}$

$\boxed{x \neq 5, -5}$

5. $\dfrac{6x^3}{2x + 4}$

$\dfrac{36x^3}{2(x+2)}$

$\dfrac{3x^3}{x+2}$

$x \neq -2$

Handwritten notes (left margin, bottom)

① SIMPLIFY THE FRACTION →

② IDENTIFY EXCLUDED VALUES →

2 QUESTIONS:

① Simplify the fraction

② Find Excluded Value(s) by look at factor form of DEN.

Example 3 *Simplify an expression by dividing out binomials*

① Simplify $\dfrac{x^2 + x - 12}{x^2 - 5x + 6}$. ② State the excluded values.

$$\dfrac{x^2 + x - 12}{x^2 - 5x + 6} = \dfrac{(x+4)(x-3)}{(x-3)(x-2)}$$

Factor and divide out common factor.

$$= \boxed{\dfrac{x+4}{x-2}}$$

Simplify.

The excluded values are __2__ and __3__.

$x - 3 = 0 \qquad x - 2 = 0$

$\boxed{x \neq 2, 3}$

Example 4 *Recognize opposites*

Simplify $\dfrac{10 + 3x - x^2}{x^2 - 25}$. State the excluded values.

$$\dfrac{10 + 3x - x^2}{x^2 - 25} = \dfrac{-x^2 + 3x + 10}{x^2 - 25}$$

① Reorder polynomials Factor numerator and denominator.

$$= \dfrac{-1(x^2 - 3x - 10)}{x^2 - 25}$$

② IF THE LEADING COEF IS NEGATIVE FACTOR OUT A -1.

$$= \dfrac{-1(x+2)(x-5)}{(x+5)(x-5)}$$

③ Completely factor

④ Divide out common factor. $(x-5)$

$$= \dfrac{-1(x+2)}{x+5} = \boxed{\dfrac{-x-2}{x+5}}$$

⑤ Simplify. means NO ( )'s

There are 2 possible Answers

$\boxed{\dfrac{-x-2}{x+5}}$ OR $\boxed{-\dfrac{x+2}{x+5}}$

↑ this is better

⑥ FIND

The excluded values are __-5__ and __5__.

LOOK AT THE DENOMINATOR $\quad x + 5 = 0 \qquad x - 5 = 0$

$\boxed{x \neq -5} \qquad \boxed{x \neq 5}$

✓ **Checkpoint** Simplify the rational expression. State the excluded values.

Simplifying

EXCLUDED VALUES →

6. $\dfrac{x^2 + 7x + 6}{x^2 + 3x - 18}$

$$\dfrac{(x+6)(x+1)}{(x+6)(x-3)} \quad \boxed{\dfrac{x+1}{x-3}}$$

$x + 6 = 0 \qquad x - 3 = 0$

$\boxed{x \neq -6} \qquad \boxed{x \neq 3}$

7. $\dfrac{4 - x^2}{x^2 + 5x - 14} = \dfrac{-x^2 + 4}{(x+7)(x-2)} = \dfrac{-1(x^2 - 4)}{(x+7)(x-2)}$

$$\dfrac{-1(x+2)(x-2)}{(x+7)(x-2)}$$

$$\boxed{\dfrac{-x-2}{x+7}}$$

$\boxed{x \neq -7, 2}$