KEY

11.2 Simplify Radical Expressions

Goal • Simplify radical expressions.

Your Notes

Radical Expression

Expression with a variable under a $\sqrt{\ }$

EX] $5\sqrt{x} + 6$

NOTE: $\boxed{x\sqrt{25} + 100}$ is NOT A RADICAL EXPRESSION. WHY?

VOCABULARY

Simplest form of a radical expression

(1) HAS NO PERFECT SQUARES (FACTORS) OTHER THAN 1 UNDER THE RADICAL EX] $\sqrt{75} = \sqrt{25 \cdot 3} = 5\sqrt{3}$

(2) NO FRACTIONS UNDER THE RADICAL EX] $\sqrt{\frac{4}{25}} = \frac{2}{5}$

(3) NO RADICALS IN THE DENOMINATOR EX] $\frac{10}{\sqrt{49}} = \frac{10}{7}$

Rationalizing the denominator IS A PROCESS OF ELIMINATING A RADICAL IN THE DENOMINATOR.

EX $\frac{2}{\sqrt{3}} \cdot \left[\frac{\sqrt{3}}{\sqrt{3}}\right] = \frac{2\sqrt{3}}{\sqrt{9}} = \frac{2\sqrt{3}}{3}$

Remember $\sqrt{n} \cdot \sqrt{n} = n$

$\frac{\sqrt{n}}{\sqrt{n}} = 1$

PRODUCT PROPERTY OF RADICALS

Words The square root of a product equals the Product of the Square Roots of the factors.

Algebra $\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$ where $a \geq 0$ and $b \geq 0$

Example $\sqrt{9x} = \sqrt{9} \cdot \sqrt{x} = \boxed{3\sqrt{x}}$

PERFECT SQUARES

1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, 169, 196, 225, ...

Example 1 *Use the product property of radicals*

Simplify $\sqrt{12x^2}$.

Solution

PSQ

$\sqrt{12x^2} = \sqrt{4 \cdot 3 \cdot x^2}$ Factor using perfect square factors.

$= \sqrt{4} \cdot \sqrt{3} \cdot \sqrt{x^2}$ ______ of radicals

$= 2 \cdot \sqrt{3} \cdot x$ Simplify.

Combine

combine under $\sqrt{\ }$

$2x\sqrt{3}$

Mental Prime factoring

$$\begin{array}{r|l} 2 & 12 \\ 2 & 6 \\ & 3 \end{array}$$

Your Notes

What is the pattern?

Simplify:

O (1) $\sqrt{x}$ ← simplified

E (2) $\sqrt{x^2}$ $\sqrt{x \cdot x} = x$

O (3) $\sqrt{x^3} = \sqrt{x^2} \cdot \sqrt{x} = x\sqrt{x}$

E (4) $\sqrt{x^4} = x^2$

O (5) $\sqrt{x^5} = \sqrt{x^2} \cdot \sqrt{x^2}\sqrt{x} = x^2\sqrt{x}$

E (6) $\sqrt{x^6} = x^3$

Conclusions

IF YOU TAKE THE SQUARE ROOT WITH VARIABLES AND

(A) THE EXPONENT IS "EVEN" THEN...

The variable is a perfect SQUARE

(B) THE EXPONENT IS "ODD" THEN...

IT WILL ALWAYS BE IN THE FORM

____ $\sqrt{x}$

↑ perfect square

Example 2 *Multiply radicals*

a. $\sqrt{8} \cdot \sqrt{2} = \sqrt{8 \cdot 2}$

$= \sqrt{16}$

$= 4$

b. $\sqrt{5x^3y} \cdot 2\sqrt{x} = 2\sqrt{5x^3y \cdot x}$

$= 2\sqrt{5x^4y}$

$= 2 \cdot \sqrt{5} \cdot \sqrt{x^4} \cdot \sqrt{y}$

$= 2x^2\sqrt{5y}$

QUOTIENT PROPERTY OF RADICALS

Words The square root of a quotient (fraction) equals the QUOTIENT of the SQUARE ROOTS of the numerator and denominator.

Algebra $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$ where $a \geq 0$ and $b > 0$

Example $\sqrt{\frac{4}{9}} = \frac{\sqrt{4}}{\sqrt{9}} = \frac{2}{3}$

Example 3 *Use the quotient property of radicals*

a. $\sqrt{\frac{11}{49}} = \frac{\sqrt{11}}{\sqrt{49}}$ (1) **Quotient property of radicals** SPLIT NUMERATOR AND DENOMINATOR

$= \boxed{\frac{\sqrt{11}}{7}}$ (2) **Simplify.**

b. $\sqrt{\frac{t^2}{36}} = \frac{\sqrt{t^2}}{\sqrt{36}}$ **Quotient property of radicals**

$= \frac{t}{6}$ **Simplify.**

Your Notes

Checkpoint Simplify the expression.

1. $\sqrt{16z^4}$ $\sqrt{16}\ \sqrt{z^4}$ $4z^2$	2. $4\sqrt{mn} \cdot \sqrt{5m}$ $4\sqrt{5m^2n}$ $4m\sqrt{5n}$	3. $\sqrt{\frac{15}{25}} = \frac{\sqrt{15}}{\sqrt{25}}$ $= \frac{\sqrt{15}}{5}$

Example 4 *Rationalize the denominator*

a. $\frac{\sqrt{2}}{\sqrt{5}} = \frac{\sqrt{2}}{\sqrt{5}} \cdot \left[\frac{\sqrt{5}}{\sqrt{5}}\right]$ ① Multiply by $\frac{\sqrt{5}}{\sqrt{5}}$. $= 1$

$= \frac{\sqrt{10}}{\sqrt{25}}$ ② Product property of radicals MULT THE TOP / MULT THE BOTTOM

$= \frac{\sqrt{10}}{5}$ ③ Simplify.

b. $\frac{1}{\sqrt{7r}} = \left[\frac{1}{\sqrt{7r}}\right] \cdot \left[\frac{\sqrt{7R}}{\sqrt{7R}}\right]$ Multiply by $\frac{\sqrt{7R}}{\sqrt{7R}}$ ← TO ELIMINATE THE RADICAL IN THE DENOMINATOR

$= \frac{\sqrt{7R}}{\sqrt{49R^2}}$ Product property of radicals

Mental step: remember $\sqrt{n} \cdot \sqrt{n} = n$

$=$ Product property of radicals So... $\sqrt{7R} \cdot \sqrt{7R} = 7R$

$= \frac{\sqrt{7R}}{7R}$ Simplify.

Combining radicals
IS similar to
Combing Like terms
Ex $7x - y + 4x + 10 = 11x - y + 10$

Your Notes

Example 5 *Add and subtract radicals*

STEP I: Simplify each radical
STEP II: Combine like radicals

a. $7\sqrt{5} - \sqrt{11} + 4\sqrt{5}$

$= 7\sqrt{5} + 4\sqrt{5} - \sqrt{11}$ — **Commutative property**

$=$ ______ — **Distributive property**

mental steps

$= 11\sqrt{5} - \sqrt{11}$ — **Simplify.** radicals by adding the Coef's of like radicals

b. $2\sqrt{2} - \sqrt{18}$

$= 2\sqrt{2} - \sqrt{9}\sqrt{2}$ — **Factor using perfect square factors.**

$= 2\sqrt{2} - 3\sqrt{2}$ — **Product property of radicals**

$= -1\sqrt{2}$ — **Simplify.**

$=$ ______ — **Distributive property**

$= -\sqrt{2}$ — **Simplify.**

Ex $2x - 3x = -1x = -x$

Checkpoint Simplify the expression.

4. $\frac{2}{\sqrt{5y}} \cdot \frac{\sqrt{5y}}{\sqrt{5y}} = \frac{2\sqrt{5y}}{\sqrt{25y^2}} = \boxed{\frac{2\sqrt{5y}}{5y}}$ Mental step $\sqrt{5y}\cdot\sqrt{5y} = 5y$	**5.** $3\sqrt{11} + 2\sqrt{44}$ $3\sqrt{11} + 2\sqrt{4}\sqrt{11}$ $3\sqrt{11} + 4\sqrt{11}$ $7\sqrt{11}$

Your Notes

Example 6 *Multiply radical expressions*

Multiply $(4 + \sqrt{3})(3 - \sqrt{3})$.

Solution

$(4 + \sqrt{3})(3 - \sqrt{3})$

mental step → $= 4 \cdot 3 + 4 \cdot -\sqrt{3} + 3 \cdot \sqrt{3} + \sqrt{3} \cdot -\sqrt{3}$ **Multiply.**

$= 12 - 4\sqrt{3} + 3\sqrt{3} - \sqrt{9}$ **Product property of radicals**

$= 12 - 4\sqrt{3} + 3\sqrt{3} - 3$ **Simplify.**

$= 9 - 1\sqrt{3} = 9 - \sqrt{3}$ **Simplify.**

Checkpoint Simplify the expression.

Distribute $\sqrt{7}$

6. $\sqrt{7}(2\sqrt{7} + \sqrt{3})$

$2\sqrt{7} \cdot \sqrt{7} + \sqrt{7}\sqrt{3} =$

$2\sqrt{49} + \sqrt{21}$

$2 \cdot 7 + \sqrt{21}$ $\quad 14 + \sqrt{21}$

7. $(3\sqrt{5} + 7)^2$ rewrite

$(3\sqrt{5} + 7)(3\sqrt{5} + 7)$

$9 \cdot 5 + 21\sqrt{5} + 21\sqrt{5} + 49$

$45 + 42\sqrt{5} + 49 = 94 + 42\sqrt{5}$

$(2x - 3)^2$

$4x^2 - 12x + 9$

Tip $(2x-3)(2x-3)$

$3x + 2$

simplified

8. $(2 + \sqrt{6})(8 - \sqrt{6})$

Simplify →

$16 - 2\sqrt{6} + 8\sqrt{6} - \sqrt{36}$

$16 + 6\sqrt{6} - 6 =$

$10 + 6\sqrt{6}$

See other NOTES for practice problems to put on back

Simplify Radical Expressions

① $\sqrt{225} = 15$ 225 is a perfect SQUARE $\sqrt{15 \cdot 15}$

② $\sqrt{x^4} = x^2$ x^4 is a perfect SQUARE $\sqrt{x^2 \cdot x^2}$

NOTE: VARIABLES WITH AN EVEN EXPONENT ARE PERFECT SQUARES

③ $\sqrt{12} = \sqrt{4 \cdot 3} = \sqrt{4}\sqrt{3} = 2\sqrt{3}$

PSQ

Variable with ODD Exponents:

2B $\sqrt{x^3} = \sqrt{x^2}\sqrt{x} = x\sqrt{x}$

④ $2\sqrt{18} = 2 \cdot \sqrt{9 \cdot 2} = 2 \cdot \sqrt{9} \cdot \sqrt{2} = 2 \cdot 3\sqrt{2} = 6\sqrt{2}$

PSQ

⑤ $\sqrt{20x^2} = \sqrt{4 \cdot 5 \cdot x^2} = \sqrt{4}\sqrt{5}\sqrt{x^2} = 2 \cdot \sqrt{5} \cdot x = 2x\sqrt{5}$

⑥ $\sqrt{81x^3y^4} = \sqrt{81 \cdot x^2 \cdot x \cdot y^4} = \sqrt{81} \cdot \sqrt{x^2} \cdot \sqrt{x} \cdot \sqrt{y^4} = 9 \cdot x \cdot \sqrt{x} \cdot y^2 = 9xy^2\sqrt{x}$

Perfect SQUARES

⑦ $\sqrt{96} = \sqrt{16}\sqrt{6} = 4\sqrt{6}$

Make a ladder

2 | 96
2 | 48
2 | 24
2 | 12
2 | 6
3

Circle pairs & multiply

$2 \cdot 2 \cdot 2 \cdot 2 = 16$

Your Notes

Example 6 *Multiply radical expressions*

Multiply $(4 + \sqrt{3})(3 - \sqrt{3})$.

Solution

$(4 + \sqrt{3})(3 - \sqrt{3})$

$= \_\_\_\_ + \_\_\_\_\_\_\_\_ + \_\_\_\_\_ + \_\_\_\_\_\_\_\_\_$ **Multiply.**

$= \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$ **Product property of radicals**

$= \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$ **Simplify.**

$= \_\_\_\_\_\_\_\_$ **Simplify.**

Checkpoint **Simplify the expression.**

6. $\sqrt{7}(2\sqrt{7} + \sqrt{3})$

7. $(3\sqrt{5} + 7)^2$

8. $(2 + \sqrt{6})(8 - \sqrt{6})$