

10.7

Interpret the Discriminant

Goal • Use the value of the discriminant.

Your Notes

WRITE THE QF

$$x = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$

VOCABULARY

Discriminant IN THE QF. THE EXPRESSION UNDER THE RADICAL ($\sqrt{\quad}$) IS CALLED THE DISCRIMINANT.

$$D = B^2 - 4AC$$

DO NOT GIVE IT BY TAKING THE $\sqrt{\quad}$!!

IT TELLS THE NUMBER OF SOLUTIONS AND THE NUMBER OF X INTERCEPTS.

Remember
Solutions
X intercepts
Zeros
ROOTS
↑
all mean the same

USING THE DISCRIMINANT OF $ax^2 + bx + c = 0$

Value of the discriminant	Number of solutions	Graph of $y = ax^2 + bx + c$
① $D = b^2 - 4ac > 0$ (+) DISCRIMINANT	$\frac{2 \text{ X INTERCEPTS AND } 2 \text{ SOLUTIONS}}$	
② $D = b^2 - 4ac = 0$ DISCRIMINANT = 0	$\frac{1 \text{ X INT AND } 1 \text{ SOLUTION}}$	
③ $D = b^2 - 4ac < 0$ (-) DISCRIMINANT	$\frac{\text{NO X INT AND NO REAL SOLUTION}}$	

Your Notes

Example 1 Use the discriminant

Equation IS IN STD FORM Discriminant

$ax^2 + bx + c = 0$ $D = b^2 - 4ac$

- $A=1$ $B=3$ $C=-2$ → a. $x^2 - 3x - 2 = 0$ $D = B^2 - 4AC = 9 - 4(1)(-2) = 17$ $D = +17$
- $A=3$ $B=0$ $C=2$ → b. $3x^2 + 2 = 0$ $D = 0 - 4(3)(2) = -24$ $D = -24$
- $A=2$ $B=8$ $C=8$ → c. $2x^2 + 8x + 8 = 0$ $D = 64 - 4(2)(8) = 0$ $D = 0$

Number of solutions

- a. 2 SOLUTIONS
- b. NO REAL SOLUTION
- c. 1 SOLUTION

Example 2 Find the number of solutions

Tell whether the equation $-2x^2 + 4x = 2$ has two solutions, one solution, or no solution.

Step 1 Write the equation in STANDARD FORM ($Ax^2 + Bx + C = 0$)

$-2x^2 + 4x = 2$ Write equation.
 $-2x^2 + 4x - 2 = 0$

$A = -2$
 $B = 4$
 $C = -2$

Step 2 Find the value of the DISCRIMINANT

$D = b^2 - 4ac = 16 - 4(-2)(-2) = 0$
 $D = 0$

The discriminant is 0, so the equation has 1 SOLUTION

✓ **Checkpoint** Tell whether the equation has two solutions, one solution, or no solution. STATE THE DISCRIMINANT.

1ST PUT IN STD FORM
 2ND IDENTIFY A, B, C
 3RD FIND DISCRIM
 $D = B^2 - 4AC$
 FINALLY TELL THE # OF SOLUTIONS

<p>1. $x^2 + 2x = 1$ $A=1$ $x^2 + 2x - 1 = 0$ $B=2$ $D = 4 - 4(1)(-1) = 8$ $D=8$ <u>2 SOLUTIONS</u></p>	<p>2. $3x^2 + 7x = -5$ $A=3$ $3x^2 + 7x + 5 = 0$ $B=7$ $D = 49 - 4(3)(5) = -11$ $D=-11$ <u>NO SOLUTION</u></p>
<p>3. $5x^2 - 6 = 0$ $A=5$ $D = 0 - 4(5)(-6)$ $B=0$ $D = 120$ $C=-6$ <u>2 SOLUTIONS</u></p>	<p>4. $-x^2 - 9 = 6x$ $A=-1$ $-x^2 - 6x - 9 = 0$ $B=-6$ $D = 36 - 4(-1)(-9) = 0$ $D=0$ $C=-9$ <u>1 SOLUTION</u></p>

Your Notes

Example 3 Find the number of x-intercepts

Find the number of x-intercepts of the graph of $y = -x^2 + 3x + 4$.

Change y to 0

Solution

Find the **DISCRIMINANT** of the equation $0 = -x^2 + 3x + 4$. $A = -1$ $B = 3$ $C = 4$

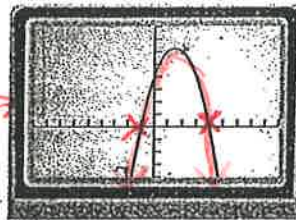
$$D = b^2 - 4ac = 9 - 4(-1)(4) = 25$$

$$D = 25$$

The discriminant is **Positive**, so the equation has **2 SOLUTIONS**. This means that the graph of $y = -x^2 + 3x + 4$ has **2** x-intercepts.

CHECK You can use a graphing calculator to check the answer.

Notice that the graph of $y = -x^2 + 3x + 4$ has **2** intercepts.



SOLUTIONS $x = -1, 4$

Checkpoint: Find the discriminant and then determine the number of x-intercepts.

5) $y = -x^2 + 3x - 3$

$$0 = -x^2 + 3x - 3$$

$$A = -1 \quad B = 3 \quad C = -3$$

$$D = 9 - 4(-1)(-3)$$

$$D = -3$$

NO X INTERCEPTS

6) $f(x) = x^2 - 4x + 4$

$$0 = x^2 - 4x + 4$$

$$D = 16 - 4(1)(4)$$

$$D = 0$$

$$A = 1 \\ B = -4 \\ C = 4$$

1 X INTERCEPT

REVIEW GRAPHING

* DESCRIBE THE SHAPE: OPENS DOWN
because $A = -1$

* WHAT IS THE Y-INTERCEPT? EXPLAIN
 $(0, 4)$ because $C = 4$

* WHAT IS THE AXIS OF SYMMETRY?

$X = \frac{-B}{2A} = \frac{-3}{2(-1)} \Rightarrow X = 1.5$ This is a line

* WHAT IS THE VERTEX? $(1.5, 6.25)$

$Y = -(1.5)^2 + 3(1.5) + 4 = 6.25$ is a point

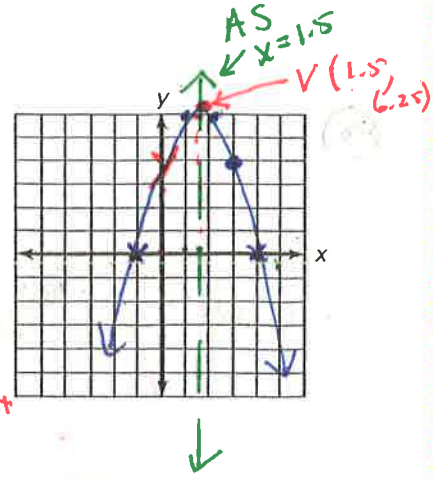
* CLEARLY PLOT 7 POINTS

TIP: CREATE A TABLE

* WHAT ARE THE X-INTERCEPTS?

$(-1, 0)$ $(4, 0)$

SOLUTIONS ARE $X = -1, 4$



EXAMPLE 3 (CONT)

$Y = -X^2 + 3X + 4$

$A = -1$
 $B = 3$
 $C = 4$

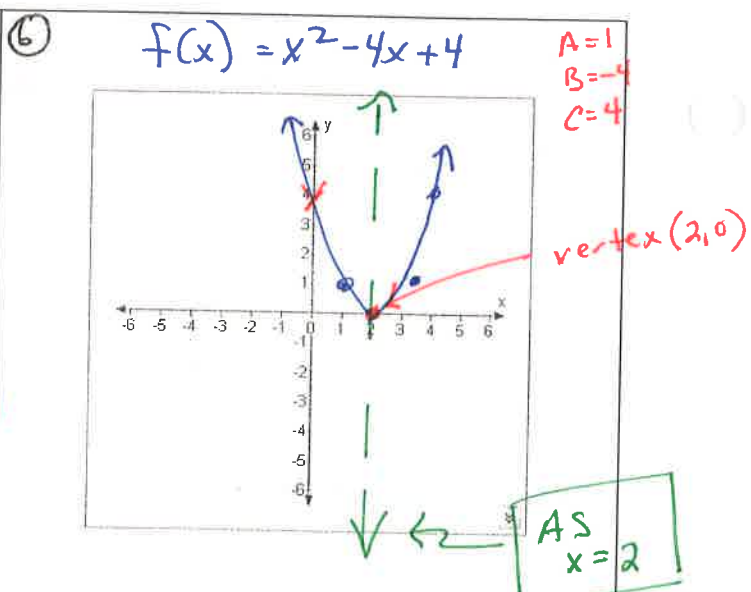
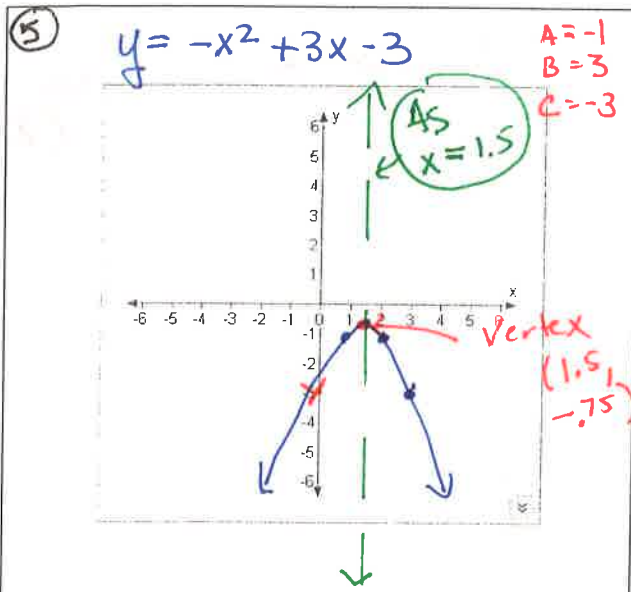
X	Y
-1	0
0	4
1	6
1.5	6.25
2	6
3	4
4	0

V

REVIEW: NOW GRAPH EACH FUNCTION

Checkpoint: (Cont.)

Instructions: Clearly plot 5 points and draw the Axis of Symmetry. Answer questions.



- ? Describe shape: OPENS DOWN ($A = -1$)
- ? Y-intercept: $(0, -3)$ ($C = -3$)
- ? A.S. $x = \frac{-B}{2A} = \frac{-3}{2(-1)} \Rightarrow X = 1.5$
- ? V: $(1.5, -0.75)$ $y = -(1.5)^2 + 3(1.5) - 3 = -0.75$
- ? X-intercept(s) NONE
- ? Solution(s) NO SOLUTION

X	0	1	1.5	2	3
Y	-3	-1	-0.75	-1	-3

- ? Describe shape: OPENS UP ($A = 1$)
- ? Y-intercept: $(0, 4)$ ($C = 4$)
- ? A.S. $x = \frac{-B}{2A} = \frac{-(-4)}{2(1)} \Rightarrow X = 2$
- ? V: $(2, 0)$ $y = 2^2 - 4(2) + 4 = 0$
- ? X-intercept(s) $(2, 0)$
- ? Solution(s) $X = 2$

X	0	1	2	3	4
Y	4	1	0	1	4