THE 5TH METHOD TO SOLVE QE'S IN THE FORM: $Ax^2 + Bx + C = 0$

10.6 Solve Quadratic Equations by the Quadratic Formula

Goal • Solve quadratic equations using the quadratic formula.

Your Notes

VOCABULARY

THE QUADRATIC FORMULA

The solutions of the quadratic equation $ax^2 + bx + c = 0$ are $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ when $a \neq 0$ and $b^2 - 4ac \geq 0$.

STANDARD Q.E.

The Discriminant and tells the # of SOLUTIONS

QUAD. FORMULA

Example 1 *Solve a quadratic equation*

Solve $2x^2 - 5 = 3x$. ← TO SOLVE WITH QF PUT INTO $Ax^2+Bx+C=0$

$2x^2 - 5 = 3x$ — Write original equation.

$-3x \quad -3x$

$2x^2 - 3x - 5 = 0$ — Write in standard form.

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ — Quadratic formula

LONGWAY

$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(2)(-5)}}{2(2)}$ — Substitute values in the quadratic formula: $a = 2$, $b = -3$, and $c = -5$.

$x = \frac{3 \pm \sqrt{9+40}}{4}$ — Simplify.

$x = \frac{3 \pm \sqrt{49}}{4}$

$x = \frac{3 \pm 7}{4}$ — Simplify the square root.

SPLIT

The solutions are $x = \frac{3+7}{4}$ and $x = \frac{3-7}{4}$

$x = 2.5$ and $x = -1$

Check your solution by graphing the related function and finding the x-intercepts.

CHECK IN ORIG EQUATION

C: $2(2.5)^2 - 5 = 3(2.5)$

$7.5 = 7.5$ ✓

C: $2(-1)^2 - 5 = 3(-1)$

$-3 = -3$ ✓

Checkpoint Complete the following exercises.

1. Use the quadratic formula to solve $2x^2 + x = 3$.

MINIMUM # OF STEPS TO SHOW:

$2x^2 + x = 3$

$-3 \quad -3$

$2x^2 + x - 3 = 0$ → write $A = 2 \quad B = 1 \quad C = -3$

Tips

① mentally take the opposite of B

② mentally/calc to FIND B^2

$$x = \frac{-1 \pm \sqrt{1 - 4(2)(-3)}}{2(2)}$$

$$x = \frac{-1 \pm \sqrt{25}}{4}$$

$+$ $\quad$ $-$

$x = \frac{-1 + 5}{4}$ $\qquad$ $x = \frac{-1 - 5}{4}$

$\boxed{x = 1}$ $\qquad$ $\boxed{x = -1.5}$

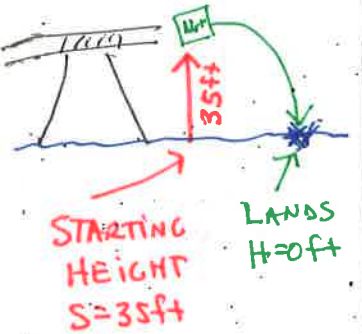
C: $3 = 3$ ✓ $\qquad$ C: $3 = 3$ ✓

Your Notes

Example 2 *Use the quadratic formula*

Crabbing A crabbing net is thrown from a bridge, which is 35 feet above the water. If the net's initial velocity is 10 feet per second, how long will it take the net to hit the water?

KI:



V = 10 ft/sec

Time = ?

Solution

The net's initial velocity is $v =$ 10 feet per second and the net's initial height is $s =$ 35 feet. The net will hit the water when the height is 0 feet.

$h = -16t^2 + vt + s$ ← Vertical motion model (memorize)

$0 = -16t^2 + 10t + 35$ Substitute for h, v, and s.

Solve

$t = \dfrac{-10 \pm \sqrt{100 - 4(-16)(35)}}{2(-16)}$ Substitute values in the quadratic formula: $a = -16$, $b = 10$, and $c = 35$.

$t = \dfrac{-10 \pm \sqrt{2340}}{-32}$ Simplify.

ROUND AT THE END!

Because time cannot be a negative number, disregard the negative solution.

$t = \dfrac{-10 + \sqrt{2340}}{-32}$ impossible $t \approx -1.99$ C: $-.04 \approx 0$

$t = \dfrac{-10 - \sqrt{2340}}{-32}$ Round to 2 Decimals $t \approx 1.82$ C: $.2 \approx 0$ (?)

NOTE: Large because of rounding error. IF we used $t \approx 1.8242$ Then C: $-.001 \approx 0$ ✓

IT TAKES ABOUT 1.82 seconds for the net to hit the water

✔ ***Checkpoint*** **Complete the following exercises.**

2. In Example 2, suppose the net was thrown with an initial velocity of 5 feet per second from a height of 20 feet. How long would it take the net to hit the water?

KI:
V = 5 ft/sec
S = 20 ft
H = 0 ft
T = ?

EQ: $0 = -16T^2 + 5T + 20$

SOLVE: $t = \dfrac{-5 \pm \sqrt{25 - 4(-16)(20)}}{2(-16)}$

A = -16
B = 5
C = 20

$t = \dfrac{-5 \pm \sqrt{1305}}{-32}$

$t = \dfrac{-5 + \sqrt{1305}}{-32}$ $t \approx -.97$

$t = \dfrac{-5 - \sqrt{1305}}{-32}$ $t \approx 1.29$

ANSWER: NET HITS THE WATER AT ABOUT 1.29 SECONDS.

Your Notes

METHODS FOR SOLVING QUADRATIC EQUATIONS

Methods	When to Use
1 Factoring	Use when a quadratic equation can be FACTORED easily. Ex: $x^2+5x+6=0$
2 Graphing	Use when approximate solutions are adequate. The solutions are the x-intercepts.
3 Finding square roots	Use when solving an equation that can be written in the form $x^2=\#$.
4 Completing the square	Can be used for any quadratic equation $ax^2+bx+c=0$ but is simplest to apply when $A=1$ and b is an EVEN number.
5 Quadratic formula	Can be used for ALL quadratic equation.

$(x+2)(x+3)=0$

$x=-2,-3$

Ex $4x^2=100$

$\sqrt{x^2}=\sqrt{25}$

$x=\pm 5$

THIS IS A DIFFICULT METHOD.

Example 3 *Choose a solution method*

Tell what method(s) you would use to solve the quadratic equation. *Explain* your choice(s).

a. $6x^2-11x+7=0$ b. $4x^2-36=0$

Solution

a. The quadratic equation CANNOT be factored easily and completing the square would result in MANY FRACTIONS. So, the equation can be solved using the QUADRATIC FORMULA.

2 ANSWERS

b. (1) The quadratic equation can be solved using SQUARE ROOTS because the equation can be written in the form $x^2=d$. or (2) This is an easy factoring problem.

$(2x-6)(2x+6)=0$

$x=-3,3$

Checkpoint **Complete the following exercise.**

3. Tell what method(s) you would use to solve $x^2+8x=9$. *Explain* your choices(s).

OPTION 1 EASY TO FACTOR

$x^2+8x-9=0$

$(x+9)(x-1)=0$

$x=1,-9$

OPTION 2 Complete the SQUARE

$(x+4)^2=9+16$

$x=1,-9$