WE HAVE A 4TH METHOD TO SOLVE QE'S

FORM QE: $X^2 + BX + C = 0$ where $A = 1$.

10.5 SOLVING QUADRATIC EQUATIONS BY COMPLETING THE SQUARE

FACTOR: $X^2+6X+9 \rightarrow (X+3)(X+3) = (X+3)^2$

INTRODUCTION TO COMPLETING THE SQUARE

I. Review **Binomial Squares.** *Think about the rules to change...*

A QUADRATIC TRINOMIAL	→	A BINOMIAL SQUARE
$x^2 + 6x + 9$	=	$(x+3)^2$

Rules: ① THE LEADING COEF (A) MUST EQUAL 1.

② THE FIRST AND LAST TERMS MUST BE PERFECT SQUARES.

③ B IS THE SQUARE ROOT OF C TIMES 2 (ie. $\sqrt{9} = 3 \cdot 2 = 6$)

II. **Completing the Square** *Now you can figure out to how to complete these squares...*

To **Complete the Square** we must ① **Calculate "C"** THEN ② **Write the Binomial Square**

$C = [\frac{1}{2}(B)]^2$

Quadratic Trinomials		Calculate "C"	Binomial Squares
$x^2 + 8x + \underline{16}$	→	$\frac{1}{2}(8) = (4)^2 = 16$	$(x+4)^2$
$x^2 - 10x + \underline{25}$	→	$\frac{1}{2}(-10) = (-5)^2 = 25$	$(x-5)^2$
$x^2 + 7x + \underline{12.25}$	→	$(\frac{1}{2})(7) = (3.5)^2 = 12.25$	$(x+3.5)^2$
$x^2 - 2.7x + \underline{1.8225}$	→	$(\frac{1}{2})(-2.7) = (-1.35)^2 = 1.8225$	$(x-1.35)^2$

III. **Here are the rules to complete the square:**

1) The coefficient of X2...must be one(1). "A" must = 1
2) Take half of the coefficient of X. 1/2B
3) Square it. $(1/2B)^2$
4) Add to the result. Do not round! The result is "C"

$(t+3)^2 = (t+3)(t+3)$

$= t^2 + 3t + 3t + 9$

$= t^2 + 6t + 9$ ← trinomial

BINOMIAL SQUARE — EQUAL

Example 2 ***Solve a quadratic equation***

Solve $t^2 + 6t = -5$ by completing the square.

Solution

$t^2 + 6t = -5$ — Write original equation.

$t^2 + 6t + \boxed{9} = -5 + \boxed{9}$ — *Add $\left(\frac{1}{2} \cdot 6\right)^2$, or 3^2 or 9, to each side.

$(t+3)^2 = -5 + 9$ — *Write left side as the square of a binomial. Check $(t+3)^2$ above

$\sqrt{(t+3)^2} = \sqrt{4}$ — *Simplify the right side.

$t + 3 = \pm 2$ — Take square roots of each side.

$-3 \quad -3$

$t = -3 \pm 2$ — Subtract 3 from each side.

The solutions — SPLIT

$t = -3 + 2$ $\quad$ $t = -3 - 2$

$t = -1$ $\quad$ $t = -5$

Check both solutions in the orig. EQ with your calc

C: $-5 = -5$ ✓ $\quad$ C: $-5 = -5$ ✓

Example 3 *Solve a quadratic equation in standard form*

Solve $m^2 - 16m + 48 = 0$ **by completing the square.**

Solution

$m^2 - 16m + 48 = 0$

Write original equation.

$-48 \quad -48$

$m^2 - 16m = -48$

(1) Subtract 48 from each side.

B

$m^2 - 16m + \boxed{64} = -48 + \boxed{64}$

CREATE A PLACE HOLDER TO COMPLETE THE SQUARE

(2) Add $\left(\frac{-16}{2}\right)^2 = (-8)^2$ or 64, to each side.

$\sqrt{(m-8)^2} = \sqrt{16}$

(3) Write left side as the square of a binomial. AND SIMPLIFY THE RIGHT SIDE

$\frac{m-8}{+8} = \frac{\pm 4}{+8}$

(4) Take square roots of each side.

$m = 8 \pm 4$

(5) Add 8 to each side.

(6) SPLIT INTO 2 EQUATIONS

The solutions are

$m = 8+4$ $\quad m = 8-4$

$m = 12$ $\quad m = 4$

C: $0 = 0$ ✓ $\quad$ C: $0 = 0$ ✓

(7) USE CALC TO Check BOTH SOLUTIONS IN THE ORIGINAL EQUATION.

To use "completing the square" method "A" must EQUAL 1

Example 4 **Solve a quadratic equation** When $A \neq 1$

Solve $5x^2 - 50x + 105 = 0$ by completing the square.

$A \neq 1$

Solution

$\frac{5x^2 - 50x + 105}{5} = \frac{0}{5}$

DIVIDE Each member (side) by 5

$x^2 - 10x + 21 = 0$
$-21 \quad\quad -21$

Subtract 21 from each side.

$x^2 - 10x + 25 = -21 + 25$

Add $\left(\frac{-10}{2}\right)^2$, or 25, to each side.

$\sqrt{(x-5)^2} = \sqrt{4}$

Write left side as the square of a binomial.

$x - 5 = \pm 2$
$+5 \quad\quad +5$

Take square roots of each side.

$x = 5 \pm 2$

Add 5 to each side.

$x = 5 + 2$
$x = 7$

$x = 5 - 2$
$x = 3$

Check all solutions in the original EQUATION

C: $x = 7$

$0 = 0$ ✓

C: $x = 3$

$0 = 0$ ✓

CHECKPOINT. Solve by completing the square

3) $X^2 - 8X = 9$

$X^2 - 8x + \boxed{16} = 9 + \boxed{16}$

$\sqrt{(x-4)^2} = \sqrt{25}$

$x - 4 = \pm 5$

$+4 \quad +4$

$X = 4 \pm 5$

$X = 4 + 5$ $\quad$ $X = 4 - 5$

$X = 9$ $\quad$ $X = -1$

C: $9 = 9$ ✓ $\quad$ C: $9 = 9$ ✓

* Remember use Calc to check in orig eq

4) $X^2 + 12X - 28 = 0$

$+28 \quad +28$

$X^2 + 12x + \boxed{36} = 28 + 36$

$\sqrt{(X+6)^2} = \sqrt{64}$

$X + 6 = \pm 8$

$-6 \quad -6$

$X = -6 \pm 8$

$X = -6 + 8$ $\quad$ $X = -6 - 8$

$X = 2$ $\quad$ $X = -14$

C: $0 = 0$ ✓ $\quad$ C: $0 = 0$ ✓

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(5) $\frac{2x^2 - 32x - 34}{2} = \frac{0}{2}$

$x^2 - 16x - 17 = 0$

$+17 \quad +17$

$x^2 - 16x + \boxed{64} = 17 + 64$

$\sqrt{(X-8)^2} = \sqrt{81}$

$X - 8 = \pm 9$

$+8 \quad +8$

$X = 8 \pm 9$

$X = 8 + 9$ $\quad$ $X = 8 - 9$

$X = 17$ $\quad$ $X = -1$

C: $0 = 0$ ✓ $\quad$ C: $0 = 0$ ✓

(22) $X^2 - 7x + 1 = 0$

$-1 \quad -1$

$x^2 - 7x + \boxed{12.25} = -1 + 12.25$

$\sqrt{(x-3.5)^2} = \sqrt{11.25}$

$X - 3.5 = \pm\sqrt{11.25}$

$+3.5 \quad +3.5$

$X = 3.5 \pm \sqrt{11.25}$

$X = 3.5 + \sqrt{11.25}$ $\quad$ $X = 3.5 - \sqrt{11.25}$

$\boxed{X \simeq 6.85}$ $\quad$ $\boxed{X \simeq 0.15}$

C: $-.0275 \simeq 0$ ✓ $\quad$ C: $-.0275 \simeq 0$ ✓