Find Slope and Rate of Change

Goal: Find the slope of a line and interpret slope as a rate of change.

Vocabulary:
- The variable that represents slope is "m".
- Slope describes the steepness of a line.
- There are 2 definitions for slope depending on whether you have a graph or 2 points.
- Rate of change compares a change in one quantity to a change in another quantity.

Finding the Slope of a Line

Method I: Given 2 points:

The slope of the nonvertical line passing through the two points \((x_1, y_1)\) and \((x_2, y_2)\):

\[ m = \frac{y_2 - y_1}{x_2 - x_1} = \text{change in } y \div \text{change in } x = \frac{\Delta y}{\Delta x} \]

Method II: Given a graph:

Graph

\[ \text{slope} = \frac{\text{Rise}}{\text{Run}} \]

Graph

\[ \text{Rise} \quad \text{Run} \]

Graph

\[ +m \quad -m \quad m=0 \quad M=\text{undefined} \]
**Example 1** Find a slope

Find the slope of the line shown. Using **rise over run**.

**METHOD I**  
Given a graph

- **M** = rise / run

- Keep the x- and y-coordinates in the same order in the numerator and denominator when calculating slope. This will help avoid error.

**METHOD II**: Use \( m = \frac{\Delta y}{\Delta x} \)

**a.** Let \( (x_1, y_1) = (-1, 2) \)  
and \( (x_2, y_2) = (3, 5) \).

\[
m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5 - 2}{3 - (-1)} = \frac{3}{4}
\]

- The line **Rises** from left to right. The slope is **Positive**.

**b.** Let \( (x_1, y_1) = (1, 4) \)  
and \( (x_2, y_2) = (3, -2) \).

\[
m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-2 - 4}{3 - 1} = \frac{-6}{2} = -3
\]

- The line **Falls** from left to right. The slope is **Negative**.

**Checkpoint** Find the slope of the line passing through the points.

<table>
<thead>
<tr>
<th>Points</th>
<th>Slope</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ((-3, -1)) and ((-2, 1))</td>
<td>( m = \frac{-1 - (-1)}{-3 + 2} = \frac{0}{-1} = 0 )</td>
</tr>
<tr>
<td>2. ((-6, 3)) and ((5, -2))</td>
<td>( m = \frac{3 - 2}{-6 - 5} = \frac{1}{-11} = -\frac{1}{11} )</td>
</tr>
</tbody>
</table>

**Remember:**

\[
-\frac{5}{11} = -\frac{5}{11} = \frac{5}{11}
\]

*These fractions are the same.*
Example 2  Find the slope of a line

Find the slope of the line shown.

- a. Let $x_1, y_1 = (2, 5)$ and $(x_2, y_2) = (-4, 5)$.
- b. Let $x_1, y_1 = (4, -2)$ and $(x_2, y_2) = (4, 3)$.

**METHOD I**

\[ m = \frac{\text{Rise}}{\text{Run}} = \frac{0}{6} \]

\[ m = 0 \]

**METHOD II** 

\[ m = \frac{\Delta y}{\Delta x} \]

- a. \[ m = \frac{5-5}{2-(-4)} = \frac{0}{6} \]
  - Substitute.
  - Simplify.
  - The line is **horizontal**. The slope is **ZERO**.

- b. \[ m = \frac{-2-3}{4-(-2)} = \frac{-5}{6} \]
  - Substitute.
  - Simplify.
  - The line is **vertical**. The slope is **undefined**.

**Checkpoint** Find the slope of the line passing through the points. Then classify the line by its slope.

<table>
<thead>
<tr>
<th>3. $(1, -2)$ and $(1, 3)$</th>
<th>4. $(-3, 7)$ and $(4, 7)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ m = -\frac{2-3}{1-1} = \frac{-5}{0} ]</td>
<td>[ m = \frac{7-7}{-3-4} = \frac{0}{-7} ]</td>
</tr>
</tbody>
</table>
Example 3  

Find a rate of change

Gas Prices  The table shows the cost of a gallon of gas for a number of days. Find the rate of change with respect to time.

<table>
<thead>
<tr>
<th>Time (days)</th>
<th>Day 1</th>
<th>Day 3</th>
<th>Day 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Price/gal ($)</td>
<td>1.99</td>
<td>2.09</td>
<td>2.19</td>
</tr>
</tbody>
</table>

Rate of change = \(\frac{\text{change in cost}}{\text{change in time}}\)

\[
\frac{\$0.10}{\text{time}} = \frac{2.09 - 1.99}{3 - 1}
\]

\[
\frac{\$0.10}{2 \text{ days}} = \frac{\$0.05}{1 \text{ day}}
\]

The rate of change in price is \(\$0.05/\text{day}\) per day.

Checkpoint

5. The table shows the change in temperature over time. Find the rate of change in degrees Fahrenheit with respect to time.

<table>
<thead>
<tr>
<th>Temperature (°F)</th>
<th>Time (hours)</th>
</tr>
</thead>
<tbody>
<tr>
<td>38</td>
<td>0</td>
</tr>
<tr>
<td>43</td>
<td>2</td>
</tr>
<tr>
<td>48</td>
<td>4</td>
</tr>
<tr>
<td>53</td>
<td>6</td>
</tr>
</tbody>
</table>

\[\text{Rate} = \frac{\Delta \text{Temp}}{\Delta \text{Time}} = \frac{43 - 38}{2 - 0} = \frac{5}{2}\]

The rate is 2.5 °F per hour.