| AP Statistics – **11.2a** | Name: |
|---|---|
| **Goal: Chi-Square ($\chi^2$) Test for Homogeneity** | Date: |

## I. Overview

- In 11.1, we used the **"$\chi^2$ Goodness of Fit Test,"** to test the null hypothesis that a categorical variable has a specific distribution (%'s). We were looking at more than 2 proportions. And used a 1-way table to organize our data.
- In Chapter 10, we learned how to compare proportions from 2 populations or treatments with the "2-sample z test for the difference of $p_1$-$p_2$."
- In Section 11.2, we will learn 2 more inference procedures to compare more than 2 samples or treatments:
  1) **$\chi^2$ Test for Homogeneity** – to compare the distributions of a single categorical variable across several populations or treatments. We will organize the data in 2-way tables.
  2) **$\chi^2$ Test for Independence**– to compare a random sample and determine if there is an association between 2 categorical variable.

## II. Organizing Data for $\chi^2$ Tests

*Does Background Music Influence What Customers Buy?*

**Comparing conditional distributions**

Market researchers suspect that background music may affect the mood and buying behavior of customers. One study in a supermarket compared three randomly assigned treatments: no music, French accordion music, and Italian string music. Under each condition, the researchers recorded the numbers of bottles of rench, Italian, and other wine purchased.[5] Here is a table that summarizes the data:

DOES BACKGROUND MUSIC AFFECT BUYING BEHAVIOR FOR WINE

3 RANDOMLY ASSIGNED TREATMENTS NO MUSIC, FRENCH, + ITAL. MUSIC

THIS WILL BE A $\chi^2$ TEST (HOMOGENEITY)

| | Music | | | | | | |
|---|---|---|---|---|---|---|---|
| Wine | None | % | French | % | Italian | % | Total |
| French | 30 | .36 | 39 | .52 | 30 | .36 | 99 |
| Italian | 11 | .13 | 1 | .01 | 19 | .23 | 31 |
| Other | 43 | .51 | 35 | .47 | 35 | .42 | 113 |
| Total | 84 | 1.00 | 75 | 1.00 | 84 | 1.00 | 243 |

% are conditional distribution for each treatment

* FOR $\chi^2$ TEST, FIRST LOOK AT DATA (a) Calculate Conditional distributions

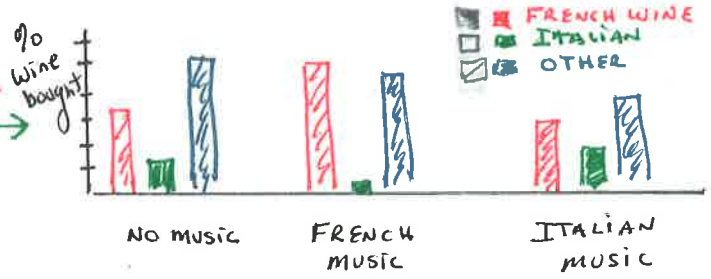
(b) Graphing will may easier to analyze data

**PROBLEM:**

**(a) Calculate the conditional distribution (in proportions) of the type of wine sold for each treatment. Display in a 2-way.**

To save time in class, add % to above table

**(b) Graph data**




**(c) Are the distributions of wine purchases under the three music treatments similar or different? Give 3 observations.**

(1) FRENCH WINE APPEARS TO BE POPULAR IN THIS MARKET AND SELLS WELL UNDER ALL 3 MUSIC CONDITIONS.

(2) SALES OF ITALIAN WINE ARE LOW WITH FRENCH (1%) + NO (13%) MUSIC AND HIGHER WITH ITALIAN (23%) MUSIC.

(3) FOR ALL 3 MUSIC CONDITIONS, THE % OF OTHER WINE IS SIMILAR.

# III. Chi-Square Test of Homogeneity – Example "Background Music"

*Does Background Music Influence What Customers Buy?*

**Computing expected counts**

The null hypothesis in the wine and music experiment is that there's no difference in the distribution of wine purchases in the store when no music, French accordion music, or Italian string music is played. To find the expected counts, we start by assuming that $H_0$ is true. We can see from the two-way table that 99 of the 243 bottles of wine bought during the study were French wines.

Observed Counts

| | Music | | | |
|---|---|---|---|---|
| Wine | None | French | Italian | Total |
| French | 30 | 39 | 30 | 99 |
| Italian | 11 | 1 | 19 | 31 |
| Other | 43 | 35 | 35 | 113 |
| Total | 84 | 75 | 84 | 243 |

EXPECTED COUNT $= \frac{(99)(84)}{243} =$ 34.22

## 1) Create a Table with the Expected Counts:

$$expected\ counts = \frac{Row\ Total \times Column\ Total}{Table\ Total}$$

Round 2 decimals.

EXPECTED COUNTS

MUSIC TREATMENTS

| WINE | NO MUSIC | FRENCH | ITALIAN | |
|---|---|---|---|---|
| FRENCH | 34.22 | 30.55 | 34.22 | 99 |
| ITALIAN | 10.72 | 9.57 | 10.72 | 31 |
| OTHER | 39.06 | 34.88 | 39.06 | 113 |
| | 84 | 75 | 84 | 243 |

## 2) Calculate the Chi-Square Statistics

a. Create 2 Lists — L1 – OBSERVED, L2 – EXPECTED
b. Create L3 with $\chi^2$ contributions (find the formula on your green sheet).
c. Place the calculated $\chi^2$ contributions in a 2-way table.
d. Add the $\chi^2$ contributions to find the total $\chi^2$ statistic

$$\chi^2 = \sum \frac{(OBS - EXP)^2}{EXPECTED}$$

L3 = (L1−L2)²/L2

$\chi^2$ CONTRIBUTIONS

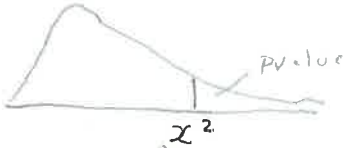
| WINE | NO MUSIC | FRENCH MUSIC | ITALIAN MUSIC | |
|---|---|---|---|---|
| FRENCH | .52 | 2.33 | .52 | |
| ITALIAN | .01 | 7.67 | 6.40 | |
| OTHER | .40 | ~0 (4.1 E-4) | .42 | |
| TOTAL | .93 | 10.00 | 7.34 | 18.27 |

TOTAL $\chi^2 = 18.27$ ← 1-Var stats L3 Σx

TIP: Check work along the way

1 VAR STATS L1 Σx = 243 ✓

1 VAR STATS L2 Σx = 243.01 ✓




## 3) State Test of Hypothesis

Ho: THERE IS NO DIFFERENCE IN THE DISTRIBUTIONS OF WINE SOLD WITH MUSIC PLAYED (NO MUSIC, FRENCH, ITALIAN)

Ha: THERE IS A DIFFERENCE IN WINE SOLD WITH TYPE OF MUSIC PLAYED

**4) Conditions**

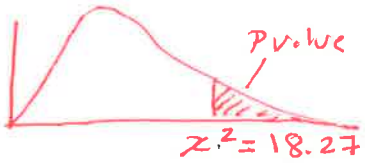
a. R ANDOM : 3 TYPES OF MUSIC (the treatment) were randomly assigned.

b. I NDEPENDENT : WE AREN'T SURE THE INDIVIDUAL OBSERVATIONS (TYPE OF WINE BOUGHT) ARE INDEPENDENT. WE MUST ASSUME THEY ARE.

c. L ARGE SAMPLE : SEE THE EXPECTED COUNT TABLE. ALL EXPECTED COUNTS ARE AT LEAST 5.

**5) Mechanics**

a. Name the test — $\chi^2$ TEST OF HOMOGENEITY
b. Sketch the graph
c. Find the degrees of freedom — $df = (\text{columns}-1)\cdot(\text{rows}-1)$
d. Find the P-value




$(3-1)(3-1) =$
$(2)(2) = 4$

$df = 4$

$$\chi^2 = \frac{(30-34.22)^2}{34.22} + \cdots + \frac{(35-39.06)^2}{39.06} = 18.27$$

To receive Full FRQ credit write this part of formula

$P(\chi^2 > 18.27) = .001$

$\chi^2 cdf(18.27, E99, 4)$

**6) Interpret the P-Value in context** make sure you know what the pvalue means!!

Pvalue=.001

Assuming that there is no difference in the actual distribution of wine purchases in this store when no music, french accordian or Italian string music is played, the probability of observed differences of wine purchases among the 3 treatment groups as large or larger than the one in this study is about 1 in 1,000 (.001).

**7) Write conclusion in context**

and small than any reasonable alpha

Since the pvalue is very small, we reject $H_0$, and conclude there is a difference in wine purchases based on the background music played

POINT TO MAKE:

THE RANDOM ASSIGNMENT ALLOWS US TO SAY THAT THE DIFFERENCE IS CAUSED BY THE MUSIC PLAYED

**8) Do a Follow-up Analysis**

SINCE WE REJECT $H_0$, we need to further investigate THE INDIVIDUAL $\chi^2$ COMPONENTS THAT CONTRIBUTED THE MOST TO THE OVERALL $\chi^2$. (LOOKING AT THE 9 COMPONENTS, ITALIAN WINE HAD THE LARGEST $\chi^2$ COMPONENTS WITH 7.6 FROM FRENCH MUSIC AND 6.4 FROM ITALIAN MUSIC.) (THESE 2 COMPONENTS TOTALED ABOUT 14 (ALMOST 77%) OF THE TOTAL $\chi^2 = 18.28$.)

→ SEE PAGE 709 - LEARN HOW TO READ COMPUTER OUTPUT, WHICH WILL MOST LIKELY BE GIVEN FOR YOU TO DO A FOLLOW UP ANALYSIS.

**9) Technology Corner**: go to page 705 and check your work using matrices and the $\chi^2$ Tests

1. ENTER OBSERVED COUNTS IN MATRIX [A] 3X3 MATRIX
2. STAT TESTS $\chi^2$ TEST OBSERVED [A] EXPECTED [B] ⟹ $\chi^2 = 18.279$ Pvalue = .001 df = 4
3. GO TO MATRIX [B] FOR EXPECTED COUNTS.

## IV. Comparing Conditional Distributions using Tables and Graphs– CYU page 698

PG698

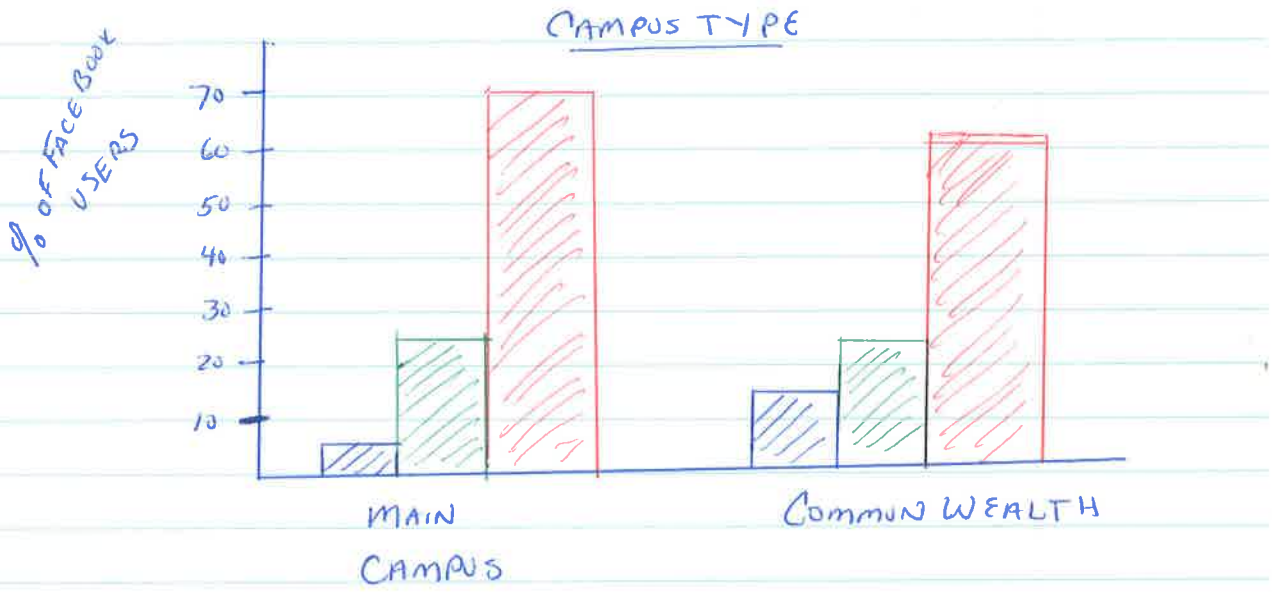
PENNSTATE

(1)

| USE FACEBOOK | MAIN CAMPUS | | COMMONWEALTH | |
|---|---|---|---|---|
| SEVERAL TIMES A MONTH | 55 | 6.0% | 76 | 12.1% |
| AT LEAST ONCE A WEEK | 215 | 23.6% | 157 | 25.0% |
| AT LEAST ONCE A DAY | 640 | 70.3% | 394 | 62.8% |
| TOTAL | 910 | 99.9% | 627 | 99.9% |

(2) YOU NEED TO COMPARE PROPORTIONS SINCE THERE IS SUCH A LARGE DIFFERENCE IN SAMPLE SIZES (910 VS. 627) FROM THE 2 CAMPUSES.

(3)




KEY:

- AT LEAST ONCE A DAY
- AT LEAST ONCE A WEEK
- SEVERAL TIMES A MONTH

## V. $\chi^2$ Test for Homogeneity from start to finish – CYU page 708

PG 708

① HEART ATTACK PATIENTS

| QUALITY OF LIFE | OBSERVED COUNTS (%'S) Canada | | U.S. | | EXPECTED COUNTS Canada | U.S. |
|---|---|---|---|---|---|---|
| MUCH BETTER | 75 | 24.1% | 541 | 25.0% | 77.37 | 538.63 |
| SOMEWHAT BETTER | 71 | 22.8% | 498 | 23.0% | 71.47 | 497.53 |
| ABOUT THE SAME | 96 | 30.9% | 779 | 36.0% | 109.91 | 765.09 |
| SOMEWHAT WORSE | 50 | 16.1% | 282 | 13.0% | 41.70 | 290.30 |
| MUCH WORSE | 19 | 6.1% | 65 | 3.0% | 10.55 | 73.45 |
| TOTAL | 311 | 100% | 2,165 | 100% | 311.00 | 2165 |

Put observed Counts in Matrix [A] and Test ($\chi^2$ Test)

Matrix B has expected Counts


② $H_0$: there is no difference in the distribution of QUALITY of life in Canada and the U.S.

$H_A$: there is a difference in the distribution of Quality of life in Canada and the U.S.

TEST: $\chi^2$ TEST OF HOMOGENEITY $\alpha = .01$ $d.f = 4 \cdot 1 = 4$

Conditions

Random – Separate random samples

Independent – We clearly have less than 10% heart attack patients

Large Sample Size – All expected counts are at least 5 (see table above)

$$\chi^2 = \frac{(O-E)^2}{E} = 11.73$$

$P(\chi^2 \geq 11.73) = .0195$


Since the pvalue(.0195) is greater than $\alpha = .01$, we fail to reject $H_0$. There is not enough evidence to conclude that there is a difference of Quality of life for heart attack patients in the US and Canada